Heading: FATIO DE DUILLIER, Nicolas (1664–1753)

Title: **Navigation improv'd** being chiefly the method for finding the latitude, at sea as well as by land, by taking any proper altitudes, together with the time between the observations: the whole made easy by a new trigonometrical rule. And the means how to refresh the air in the hold of a ship or of a man of war: and how to measure the number of waves, that pass under a ship during any given time; in order tomake a truer estimation of the ship's way: and to have likewise thereby a sort of time-keeper.

Imprint: **London**: printed in the months of April and May, 1728

Collation: Folio: [pi]^2 ([pi]1 + A–B^2), 6 leaves, pp. [4] 5–12. Author's manuscript note dated 5 June 1728 on verso of titlepage (see below).

Plates: 1 folding engraved plate.

Condition: 359 x 232mm. Some light dustoiling; plate waterstained.

Binding: Disbound; evidence of original stab-stitching.

Provenance: Probably William Jones (1675–1749) with annotations which appear to be in his hand; Earls of Macclesfield.

References: ESTC T58837.

Price: **£3,500**

Edition: First (only) edition.

Fatio's method of finding latitudes, and other improvements to navigation, written up in 1716 after successful trials at sea but not previously published. Evidently a private publication (with no publisher's imprint) this was probably only presented to a small number of people and the British Library copy, like the present copy, has a note in Fatio's hand on the verso of the titlepage offering to demonstrate his method at sea. In the present copy this reads: 'London June the 5th 1728. I shall be further willing, upon proper Encouragement, to go to Sea a short Voyage to demonstrate and teach the Method which I do here propose; As, for instance, a voyage to Gottenburg, Copenhagen, or Portugal. N. Facio.'

Fatio was a close friend to Newton and was involved in the Newton–Leibniz dispute over priority in the invention of the calculus in which William Jones was also a key player. This copy must have been given to Jones by Fatio. Jones' was tutor to the Earl of Macclesfield to whom he left his mathematical books.

In the preface, Fatio mentions his invention of jewelled bearings for watches for which he was awarded a patent in 1704, which he was forced to abandon because of opposition from the clockmakers company. The method of refreshing the air between decks is also briefly mentioned in the preface but not in the text.

Apart from the method of determining latitude from extra-meridian

altitude and elapsed time, the text deals with a method of measuring the ship's way from the number of wave crests passing under the ship in a given time and their wavelength.

ESTC records copies at the British Library, Bodleian Library, Royal Society and New York Public Library. Not on ECCO.

Literature: Taylor, *Mathematical practitioners of Hanoverian England* pp. 117–8; Scott Mandelbrote in *ODNB*.

NAVIGATION IMPROV'D:

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In order to make a truer *Eſtimation* of the Ship's Way:

AND

To have likewiſe thereby a ſort of Time-keeper.

By NICOLAS FACIO DUILLIER, *R. S. S.*

LONDON:
Printed in the Months of April *and* May, 1728.

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I shal be further willing, ~~to~~ ~~go~~ upon proper Encouragement, to go to Sea a short Voyage, to demonstrate and teach the Method which I do here propose; as, for instance, a Voyage to Gottenburg, Copenhagen, or Portugal.

N. Facio

PREFACE.

AMONG the many useful things, which Providence and my long and too unfortunate and ruinous Study of the Mathematicks has enabled me to find for the Improvement and Safety of Navigation, I shall chuse to publish now chiefly one, and to point out at some few more: By which I shall invite the Generosity and Justice of this flourishing Nation, in pursuance of the Acts of Parliament that are in force, to demonstrate, if they think fit, how far they were in earnest when those Promises and Invitations were made, which make me now to apply in this manner to the Public.

Had not I buried, in consequence of those Inquiries, by far the best part of my Substance, especially in introducing and establishing of Jewel-Watches, through all manner of Difficulties and Oppositions; for if I knew that Common Watches might do, I knew also that Jewel-Watches would do better; Surely I had, with the greatest Pleasure imaginable, published early and freely what I knew would be of general Advantage. But as I thought it would be but just, that, if the Public reaped the great Benefit arising from what I had found in this kind, I should, at least, be no loser by it; I confess I have only, from time to time, made, during these last five and twenty Years, some Attempts to try whether, after so great a Charge in sowing too freely, I could be sure of a Harvest, or of recovering the Losses and Damages which I had incurred, or at least a part of them, if I communicated or published the whole. But I found nothing that promised any Certainty of my being countenanced or encouraged any ways; but sometimes, and too often, quite the contrary. A melancholy Scene for one who reckons himself now the worse at least by three thousand Pounds, for having applied himself to those very Inquiries, and that too with uncommon Success!

However, seeing no Hopes of compassing the public Ends which I proposed to myself, unless it be done in my life-time: This requires I should now make haste, and I set about it as follows.

I shall just hint, in the first place, what I found very lately, to wit, That there is an easy Method for refreshing continually the Air in the Hold and between Decks, in a Ship at Sea, by one or more large Bellows or Air Pumps playing continually by the force of the Ships rowling, or dipping, or ballancing itself any way. By the like Method, one or more Pumps, nay even single Canals or Tubes, may work of themselves, for pumping the Water out of a Ship. How many Lives might thus have been spared, even during these few last Years, let them judge that understand Physick, or have been, for instance, in the Royal Fleet, between the Tropicks for some Years. And thus much is enough for discovering the thing in general. But the Execution of it I leave to every ones own Choice and Conveniency.

Secondly, I shall just mention a Method for finding at Sea, by Observation, the Measure of the various Swiftness and Bigness of Waves. The same may be found with some degree of Exactness, by a just Theory also. And, if this be done, or even before it be done, some considerable Advantages will arise from an Instrument shewing continually upon a Dial Plate, by different Needles, or by Holes in the Plate, how many Thousands, and Hundreds, and Tens, and Units of Waves have passed under the Ship during any given Interval of Time, suppose an Hour or a Day &c, the Obliquity of the Course of the Ship to the Waves being observed also. Now that Interval of Time will be known, by any good Common or Jewel-Watch.

The Theory of Waves, as to their Bigness and Swiftness, is exceeding difficult to be found. It has escaped the Industry and Sagacity of Sir Isaac Newton *himself, tho' he has handled expresly this Matter: For he makes the Waves much smaller than I have found them, both by actual Observations at Sea, and by a Theory grounded upon some truer Principles, which that great Man did overlook. But this may be treated of more fully in another Place, together with some other things relating to the Perfection of Navigation: And then will be a properer time to write those minute Directions, that may serve best for mean Capacities. But now my chief Business is to convince Them that are more learned, and Them also who, if they but please, can want neither Power nor Methods, to bring these Inventions into use:*

Waves, if they be once settled, are exceeding regular; and, while they continue of the same Bigness, they may even serve for a good Time-keeper. Upon which account, if in a Ship or Vessel there be no Watch or Clock, yet sometimes the Meridian Altitude of the Sun may be found on such Days as the Sun is seen but now and then between the Clouds, though there be no possibility of coming then by a Meridian Observation.

Let Q,R,S, *Fig. VII. Represent in a large Abacus, or graduated Frame or Instrument, three or more Places in the heavenly Sphere, where the Sun's Altitude was observed: But let not those Places be too far from Noon. Therefore, upon the Horizontal Line* QTV, *take or conceive the Segments* QT, TV *proportional to the Number of Waves that have passed under the Ship between the Observations. Make* TR *and* VS *perpendicular to* QTV. *Through the Points* Q,R,S, *&c. draw the uniform Curve or the Arc* QRS. *The remotest Point in the Curve* QRS, *from the Basis of the Abacus, shall give the Sun's Meridian Altitude for that Day. This Abacus may be engraved on purpose on a Board or a Slate, for these and the like Operations.*

This is a Paradox indeed, that some Altitudes of the Sun, taken off the Meridian, should, without a Watch or Clock, enable us to know the Sun's Meridian Altitude: But the Thing is plain, and the Practise easy, and not above the meanest Comprehension.

The great Number of Ships and other Vessels, and of Mens Lives also, that may be saved by these very Methods, in twenty or a hundred Years, shall be a Witness before Heaven and Earth, that will speak for me and my Executors, in case I alone should remain a Loser, whilst Providence makes use of me, to procure so much Good to others.

INTRODUCTION.

SOON after Pendulum Watches and Clocks were invented, some Mathematicians did perceive that the Latitude might be found at Land, by observing two convenient Altitudes of the Sun, or of the Sun and a Star, or of two Stars, together with the Time between both Observations. I think I remember to have seen in the Works of Sir *Jonas More* an Idea of this Problem; and that, because of the Usefulness of it, and for other Reasons, he printed a Table of the Versed Sines both Natural and Logarithmic, in order to facilitate the Trigonometrical Calculation, which the Solution of this Problem and of some others also requires.

But the Trigonometrical Rule which he gives for these Calculations, being, as I suppose, the very same which Mr. *Sherwin* gives in his Mathematical Tables, among which he has published also the Table of the Natural and Logarithmic Versed Sines; and that Rule being clogged by those Authors, and not given in its Natural Simplicity, nor very easy to be remembred; we do not perceive that hitherto it has been made use of for the Advancement and Perfection of Geography, and much less for the Perfection of Navigation. Nay, describing the Uses of that Trigonometrical Rule, Mr. *Sherwin* expresses himself only in the following Words:

" This *Proposition* is of great Use to calculate the " Distance of Places on Earth, according to the *Arch* " of a great Circle, by their *Longitude* and *Latitude* " given; the Distances of Stars, by having their De- " clinations and Right Ascensions, or Longitudes and " Latitudes given: By means whereof the Altitudes " of two Stars, or of the Sun, with the Difference " of Time or *Azimuth* being observ'd at any Time off " the *Meridian*, the Latitude may be found". In these Words there appear no footsteps of any Pretensions, and much less of any Method to find by these *Data* the Latitude of Ships under Sail; whereas I shall give a full Solution of this most useful Problem, though a Ship should alter its Latitude even by a hundred Miles or more, between both Observations. And I shall confirm that Solution by a notable former Experience, which may soon be followed by many more.

N. B. *The Intricacy of the said Trigonometrical Rule printed by Mr.* Sherwin *is such, that in the two Examples given by him he has committed several Mistakes. In the use of that Rule he must write down eight Lines of Calculation, whereas I write but six. And besides he must take the half of a given Arc, and write down the double of a Logarithmic Sine, instead of my writing the Numbers as they are actually given, or else as they are found in the Tables. By which means I spare much Trouble, and am less exposed to Numerical Mistakes, and have sooner done; which may be sometimes of the utmost Importance at Sea.*

However the said Trigonometrical Rule agrees in effect, by a round-about-way, with the Rule which I found a great many Years ago: Which last Rule makes part of the *Latin* Manuscript written by me in 1716, for the Improvement of Navigation; and is as follows. Neither did I perceive the near Agreement of those two Rules, till the end of the Year 1726.

TRIGONOMETRICAL PROPOSITION.

In this Projection of the Sphere ADPEFA, *Fig. I. which supposes the Eye at an infinite Distance, let* APB *be any Spherical Triangle proposed. Take the Arcs* PD, PE, *equal each of them to the Side* PB. *Draw the Chord* DBE. *Draw* DG *parallel to the Diameter* AHICF; *and draw the Lines* DH, BGI *perpendicular to the same.* Fig. 1

The Radius | is to the Versed Sine of the Angle APB || as the Sine of DP or of PB | is to DB.

So then $DB = \frac{\textit{Vers. Sin.}\text{ of the Angle } APB \times \textit{Sine}\text{ of } DP}{\textit{Rad.}}$

Rad. AC | is to the *Sine* of AP || as DB found above | is to DG or $HI = \frac{\textit{Sin.}\,AP \times \textit{Sin.}\,DP \times \textit{Vers. Sin.}\,APB}{\textit{Rad. quadr.}}$

From this last Analogy I derive the following Rule.

Two Sides of a Spherical Triangle being given, and the Angle between them, to find the third Side.

SOLUTION.

To the Logarithm of the Versed Sine of the given Angle, add the two Logarithmic Sines of the two Sides; and from the Sum cut off twice the Radius. Then seek the Natural Number of the remaining Logarithm. (That Number will give you the Length of HI.) To which Natural Number add the Natural Versed Sine (AH) of the Difference of the Sides. The Sum will give the Natural Versed Sine (AI) of the third Side or of the Base (AB.)

By the same Rule, *Having the Three Sides of a Spherical Triangle, you may find the Angle opposed to that Side which you chuse to call your Base.*

If for any Angle, for instance the greater Angle, and for its opposite Side, we take their Complements to 180 Degrees; the Angles shall be turn'd into Sides, and the Sides into Angles. And by consequence the Trigonometrical Proposition here demonstrated will serve for the Two following Cases also.

In a Spherical Triangle, *Two Angles being given, and the Side between them; to find the third Angle.*

And likewise, *Three Angles being given, to find the Side opposite to which Angle you please.*

If a Natural Versed Sine exceed the Radius; from the same cut off the Radius, and the Remainder will be the Natural Sine of the Excess of the Angle or Arc above 90 Degrees. If an Arc exceed 90°, its Natural Vers. Sine is the Radius added to the Natural Sine of the Excess of the Arc above 90°.

A Logarithmic Versed Sine that exceeds the Radius answers to a Natural Number, from which if you cut off the Radius, the Remainder is the Natural Sine of the Excess of the Arc or Angle above 90 Degrees.

If you continue the Side AB to a Semicircle ABF, you may chuse to resolve either the Triangle APB, or in its stead the Triangle PBF; for the Solution of the one involves the Solution of the other. Therefore chuse that which may be more easily solved.

There is a considerable Advantage in Trigonometrical Calculations, if one uses Natural Versed Sines, and their Logarithms, rather than Natural Sines and their Logarithms. But of this I may speak elsewhere.

PRO-

PROBLEM.

To find the Latitude, *by Two Altitudes of the Sun and the Time between them; supposing the Observations to have been made from the same Place, or from different Places, whether at Sea or Land. And likewise to find the Hours, and the Sun's Azimuths, at the Times of both Observations.*

SOLUTION.

EXAMPLE I.

Fig. 2. SUPPOSE that in 1727. *Nov.* 3. O.S. in the Afternoon, the Sun being in A, Fig. II. its Distance A P from the North Pole P being 108° 13′, you observed the Distance A Z, from the Zenith Z, after allowing for the Refraction occasioned by the Atmosphere, to have been 75° 14′ when your Watch or Clock did shew - - - - - - - - - - - - - - $2^h 25^m 22^s$

And that *Nov.* 7. in the Morning, the Sun being in B, its Distance B P from the North Pole P being 109° 12′, you observed likewise its Distance B Z from the Zenith to have been 77°. 41′ clear of Refraction, when the Hour by the Watches was such that, upon the whole, you conclude the Angle A P B, contained between the Sun's Horary Circles P A and P B, did answer to $5^h 21' 20''$ of Time, or to 80° 20′. And this must be after allowing for the Equation 42″ of Time, being so much as a Watch fitted to the Sun's mean Motion ought then to go slower than the Sun in 3¾ Days; and after allowing also proportionably for what you know or do suppose your Watches did get or lose every Day upon the Sun's mean Motion.

I speak here of Watches in the Plural Number, because it will be safest at Sea to have at least three of them. Major *Holmes* had three Watches, when he made those notable Experiments related in the Philosophical Transactions. If you have but two, the one may have some Irregularities, which could not be discovered so well by its differing from a second Watch. But the concurring Witness of two or more Watches against it, discovers better the Error, or at least gives the Hour surer, by taking a Medium between them all.

I. In the Triangle A P B, I find the Side A B by my Rule demonstrated heretofore; which I do as follows:

Log. Vers. Sin. APB 80.20. 992017
Sinus P B 109.12. 997515
Sinus PA 108.13. 997767
2987299 Log. Num. 74643
Nat. Vers. Sin. of the Dif. of the Sides 0.59. (6. being Characteristick 15
Nat. Vers. Sin. of AB = 75.19⅓ 74658

$v.c = \frac{S.A \times S.B \times v.c}{rr}$ +

II. The Angle P B A might be found by the Sines of the Angles being proportional to the Sines of their opposite Sides. But this Proportion leaving you to seek whether the Angle sought be greater or smaller than 90°, and therefore exposing you sometimes to mistake the Supplement to 180° instead of the Angle it self, it will be safest to use the preceeding Rule; and this must be done as follows.

Q. the V^s. Sin. of PBA? 1009724 } L. of 125093 which is the V.S. of 104°. 32′ = PBA.
Sin. PB 109.12. 997515
Sin. BA 75.19⅓. 998559
Log. 1005798 114283
Nat. Vers. Sin. of the Dif. of the Sides 33.52⅔ (9 Char.) 16978
Nat. Ver. Sin. of P A = Rad. + Na. Si. of 18°.13′ = 131261

$v.c = \frac{v.C - v.X}{S.A \times S.B} \times rr$

III. The Angle ABZ is found thus, and also the Angle P B Z.

Qu. the Vers^s. Sin. of ABZ? 989625 V. Si. of 77.44 = ABZ
Sin. B Z 77.41 998989 104.22 = ABP
Sin. A B 75.19⅓ 998559 26.48 = PBZ.
987173 74427
Na. Ver. Si. of the Dif. 2.21⅔ (6 Char. 85
Nat. Ver. Sin. of AZ 75.14 (9 Char. 74512

IV. Now to find P Z work thus.

Si. Ver. PBZ, 26°48′ 903106
Sin. PB, 109. 12 997515
Sin. BZ, 77. 41 998989
8.99610 99106
N.S.V. Dif. 31. 31 (9 Char. 14751
24662 Si. V. PZ = 41°. 7.
So then the Latitude of the Point Z is 48. 53.

Wherefore these four short Operations, all perform'd by our single and most easy Rule, would give you the Latitude of the Place 48°. 53′. if the Observations were made by Land, or the Ship had remain'd unmoveable, or was return'd to the same Place again. It is to be hoped that the great Facility of this Method will invite many Travellers by Land, to take this obvious means of rectifying our too defective Geography.

V. To find the Angle BPZ, and by consequence the Angle APZ also, or the Hours answering those Angles, work thus:

80.20 = APB
Qu. Sin. Vers. B P Z? 941087 Sin. Vers. 42. 3½ = B P Z answering $2^h 48^m 14^s$ Distance from Noon.
Sin. P B, 109.12 997515
Sin. P Z, 41. 7 981796 38.16½ = A P Z answering $2^h 33^m 6^s$ after Noon.
920398 L. of 15995
Sin. Vers. Diff. 68. 5 (9 Char. 62674
Nat. Vers. Sin. BZ, 77° 41′ 78669

Through the Point Z draw the Circles or Arcs Z S and Z Q, whose Poles are A and B.

The same Rule by which those five preceeding Operations were made, may serve likewise to find the Azimuthal Angles P Z A, P Z B; especially if any of those Angles at Z be so far uncertain, that you are not sure whether it be bigger or smaller than a Right Angle. Otherwise you may use the Proportion betweeen the Sines of the Sides and the Sines of their opposite Angles, as I do here; the Calculation being a little shorter.

VII. To find P Z A.

Sin. A Z, 75.14 0.01459 Comp. Ar.
Sin. APZ, 38.16½ 9.79200
Sin. AP, 108.13 9.97767
Sin. 37.29 9.78426
Suppl. 142.31 = P Z A. P Z S = 52°. 31′.

VI. To find P Z B.

Sin. B Z, 77.41. 0.01011 Comp. Ar.
Sin. B P Z, 42. 3½ 9.82600
Sin. P B, 109.12. 9.97515
Sin. 40.21 9.81126
139.39 Suppl. = P Z B. P Z Q = 49.39.

VIII. In order to verify this whole Calculation, I find the Angle A Z B by our former Rule.

Quær. A Z B? 9.89726 Si. Ver. 77°.50 = A Z B.
Sin. B Z, 77.41 9 98989 139.39 = P Z B.
Sin. A Z, 75.14 9 98541 142.31 = P Z A.
9 87256 74569 360. 0 This Sum confirms the whole Calculation to be right.
Si. V. Na. 2.27 6 Char. 91
Si. V. Na. A B, 75.19⅓ 74660

Thus

Thus those eight Calculations may be perform'd by the Help of our single Rule: To which you may intermix if you please, as I have done, that most easy but equivocal Rule, that the Sines of the Sides are proportional to the Sines of their opposite Angles. For this last Rule may serve to make the II. V. VI. VII and VIII Calculations. You shall see hereafter the great Use of this Preparation, even for finding the Latitude at Sea.

If two Persons that understand this Method help one another in their Calculation, the one seeking the Numbers in the Table while the other does write, the Work may be done in half the time, and with more Certainty; since the one will prevent the Mistakes which the other by Inadvertency might fall into. Many other Abridgments might be made use of, both for Ease and Diligence. For instance, some Schemes may be printed, or else engraven upon a Slate, with the whole Process of the Calculation, wanting only the particular Numbers to be inserted each in its proper Place: For thus the trouble of so much thinking, and writing, and verifying, and of making a new Scheme, will be spared; and the Danger of falling into Mistakes will be very much diminished. Now in some ticklish Circumstances of many sorts, it may be of the utmost Consequence at Sea to gain Time, to alter as soon as may be the Course of a Ship, or even of a Fleet.

Example II.

Suppose, now, that, having made at Sea the very same Observations of the Sun, and at the very same Days and Hours, as in the first Example, your Ship, in the mean time between the Observations, did sail almost a South South East Course, so that you conclude, upon the whole, its Course did amount to an Arc of a great Circle of 156 Minutes or Miles in length, and did make at first an Angle of 159° with the Meridian.

It is asked which were both your Latitudes, at the Times of the Observations.

Which were the true Hours of the Day, at the Times and Places of both Observations.

And what Difference of Longitude results between both Places of Observations.

This Example as well as Example III is offered here with great Disadvantage: For, here is no mention of any the like anteriour Observations, by which the Latitude of
3 the Point S (Fig. III) might be better ascertained. The Interval of Time between the Observations is very great; and so is likewise the Run of the Ship between the Observations. Lastly, the Arcs SZ and ZQ cut each of them the Meridian PZ with a great Obliquity; whereas it would be much better if one of those Arcs, and especially the Arc ZQ as belonging to the Second Observation, did cut the Meridian as near as might be at Right Angles.

When so many Difficulties occur, a ready Calculator will not grudge beginning his Calculation over again, after having made such Alteration in his uncertain *Data*, as his Conclusions will best direct him to. But he must be so much the more careful in procuring an Altitude nearer to the Meridian as soon as may be; or at least two Altitudes not separated by so long an Interval of Time.

To resolve this useful Problem I take the following Steps.

I. I make the very same Suppositions, and the very same seven or eight Calculations as in the first Example, in order to find not only the Dormant or Borrowed Zenith Z, which would have resulted from the Observations, if the Observator had been at rest; but likewise all the Arcs and Angles which are given or found by those very Suppositions and Calculations.

II. By this means, supposing the little Triangle XZY (in the III Figure) to result from those Calculations, and to be a Rectilinear Triangle, since it differs but little from one; and supposing also the Meridians passing thro' the several Points X, Y and Z to be sensibly parallel; and XY = 156 Minutes or Miles to represent your Course: You have all the Angles in the Triangle XZY, namely, the Angle at X of 31° 31'; the Angle at Z of 77°. 50'; and the Angle at Y of 70°. 39'. Fig. 3

Thus calculating the Side ZX, or else construing the Figure it self, you will have with no great Error the Point X for the Place of your Ship, at the Time of the first Observation.

III. This however is but a very near Approximation, which you may correct either by a mechanical Construction, or rather by a further Calculation, as follows; or, if you will, by both.

As your Aproximation may best direct you, take two different Values of ZX or Zs; the Difference between them being ten Minutes, or, if you will, twenty Minutes: And for these two Values find severally in the Triangles PsZ their Sides Ps.

IV. Then, for the two Values of Ps, find in the same Triangles PsZ their Angles sPZ.

V. Then, for the two Values of Ps, find, in the Triangles Psq, the two Values of Pq.

VI. Then in the same Triangles Psq find the two Values of the two Angles sPq. And so you will have the two Angles qPZ likewise.

VII. Then, in the two Triangles qPB, find the two Values of qB.

VIII. Now QB being given, as here of 77° 41', its Differences from both the Values of qB will be known. And if you work proportionably on the respective Differences of both the Values of Ps, sPZ, Pq, sPq and qPZ, you will find the true corrected Values of PS, SPZ, PQ, SPQ and QPZ; and by consequence you will have the full Resolution of the present Problem.

I. All the Work of the first Step or Article is already dispatch'd in the first Example.

II. The Side ZX in the Triangle ZXY will be found by this Proportion.

Sin. 77. 50 | Sin. 70.39 || 156 Miles | { 150'. 57 Miles = ZX = 2°. 30' 34".

Comp. Ar. Sin.	77.50.	0.00987	
Sin.	70.39.	9.97475	
Log.	156.	2.19312	
		2.17774	Log. 150. 57.

III. Let the two Values of Zs be taken, the one = 2°.22'. And the other Value of Zs = 2°.32'.

So then, in the Triangles PsZ, I find the Sides Ps as follows.

Sin. Verf. PZs,	52.31	959270		
Sin. PZ,	41. 7	981796		
Sin. sZ,	2.22	861589		
		802655	Log.	1063
Si. Ve. Na. Dif.	38.45	(9 Ch.)		23012
				23075 39° 43' = Ps.

Sin. Verf. PZs,	52.31.	959270		
Sin. PZ,	41. 7	981796		
Sin. sZ,	2.32	864543		
		805609	Log.	1138
	38.35	(9 Ch.)		21830
				22968 39° 37' = Ps.

PS = 39.39½

IV. To

IV. To find the Value of the first Angles P s Z and s PZ.

Sin. Ps | S.sZP || Sin. sZ = 2.22 | Sin. sPZ = 2 . 56 2/5.
39.43 | 52.31 || Sin. PZ = 41. 7 | Sin. PsZ = 125 . 15.

Com. ar. Sin. 39.43.0.19450
Sin. 52.31.9.89956
Sin. 41. 7.9.81796
Sin. 54.45.9.91202

Comp. ar. Sin. 39.43.0.19450
Sin. 52.31.9.89956
Sin. 2.22.8.61589
Sin. 2.56 2/5 8.70995

P s Z = 125.15 SPZ = 3°4′ 1/5

To find the Value of the second Angles P s Z and s PZ.

Sin. Ps | Sin. s ZP || Sin. s Z = 2.32 | Sin. s P Z = 3.9 1/5
39.37 | 52.31 || Sin. PZ = 41.7 | Sin. P s Z = 125.5

Co. ar. S. 39.37.0.19542
S. 52.31.9.89956
S. 2.32 8.64543
8.74041 Si. 3°.9 1/5

39.37.0.19542
52.31.9.89956
Sin. 41. 7.9.81796
9.91294 S. 54.55
PsZ = 125. 5
P s q = 159.
Z s q = 33.55

V. To find P q in the first Triangle P s q.

Sin. Verf. 159° 1 028636.19.3358
Sin. P s 39.43.980550
Sin. s q 2. 36.865670
874856 56048
Sin. Verf. Nat. 37. 7(9) 20259
Sin. Verf. P q = 42.9 25864
—3 1/5
PQ = 42 . 5 2/5

To find P q in the second Triangle P s q.

Sin. Verf. 159° 1028636
Sin. P s 39. 37. 980458
Sin. s q 2. 36. 865670
874764 55929
Sin. Verf. Nat. 37. 1.(9) 20154
Sin. Verf. P q = 42.3 1/20 25747

VI. To find the Angle s P q in the first Triangle P s q.

Sin. P q, 42°.9′ | Sin. P s q, 159° || Sin. s q, 2°.36′ | Sin. s P q.
Sin. 42.9 . 0.17323 1.23 1/5
Sin. 21. 9.55433
Sin. 2.36 8.65670 2.56 2/5 = s PZ
8.38426 Sin. 1.23 1/5 = s P q 1.23 1/5 = SPQ
1.33 3/20 = q PZ
But Z P B was found 42. 3 1/2 = Z P B
43.36 2/5 = q P B 43.44 1/2 = QPB
7 1/5 Correction.

To find the Angle s P q in the second Triangle P s q.

Sin. P q, 42.3 1/20 | Sin. P s q, 159° || Sin. s q, 2°.36 | Sin. s P q.
Sin. 42.3 1/20 0.17386
9.55433
8.65670 3. 9 1/5 = s PZ
8.38489 Sin. 1.23 2/5 = s P q
1.45 4/5 = q PZ
42. 3 1/2 = ZPB
43.49 1/10 = q P B Diff. 12,633.

VII. To find q B in the first Triangle q P B.

Sin. Verf. 43.36 2/5 944085
Sin. P B, 109.12 997515
Sin. P q, 42. 9 982677
924277. 17489
67. 3 61007
q B = 77.35. 78496. Instead of 78668 1/2,
Which is the Natural Versed Sine of QB = 77.41

To find q B in the second Triangle q P B.

Sin. Verf. 43.49 1/10 944476
Sin. 109.12 997515
Sin. P q, 42. 3 1/20 982595
924586 17614
67. 8 1/20 61164 1/2
q.B = 77.44 6/7 78778 1/2

78778 78668 1/2
78496 78496
282 172 1/2

$\frac{345}{564} = \frac{115}{188} = 0,6117$

In this Proportion must all the Differences be corrected.

If from A P Z = 38°.16 1/2 you take S P Z = 3°.3 4/5, you will have A P S = 35.12 7/10 answering to the true Hour in S when the first Observation was made; *viz.* 2^h 20^m 51^{sec} in the Afternoon.

The difference of Longitude between the Points S and Q is 1°23′ 1/5 answering to 5^{min} 33^{sec} of Time.

And the Angle Q P B = 43°44′ answers to the true Hour in Q when the second Observation was made, *viz.* 2^h 54^m 56^s. before it was Noon.

Now whereas in the Points S and Q you did set down the Hours by your Watches, at the Times of the first and the second Observations, which otherwise would be but lame; if these Hours given by the Watches be compar'd with the true Hours of the Sun as determined here, having a due regard to the Equation of the Sun, and to the Swiftness of your Watches in 24 Hours, and to the Places where your Watches were compared last with the Heavens: You will have all the Lights that these Observations can give you, in reference to the Longitude of the Points S and Q, and also to the Swiftness of your Watches, and the Length and Position of SQ.

Example II, Reduced to a greater Simplicity.

If, rather than to be at the trouble of making so full a Calculation, you chuse to seek only your Latitudes, or else the Distances P S and P Q from the Pole, by an Approximation erring but a few Minutes at most; you may, from the Value of Z X (N°. 2,) calculate (instead of N°. 3) the Value of PX, which will be found 39°37′ as follows, differing but 2 1/2 Minutes from the corrected Value of PS.

Sin. Verf. P Z X 52.31. 959270
Sin. 41. 7. 981796
Sin. 2.34.5.865133
806199 Log. 11534
38.32.55 21810
22963. 39°.37′ = P X.

Then to find the Angle P X Z, you may say;

Sin. P X | Sin. P Z X || Sin. PZ. | Sin. PXZ = (54.55)
39.37 52.31 41.7 125. 5
Co. ar. Sin. 39.37 0.19542 159. = P X q
52.31 9 89956 33.55 = Z X q
41. 7 9 81796 52.31 = P Z X
9.91294 18.36 Inclination of X q to the Meridian PZ.

And so having the Inclination of X q to the Right Line Z X, you will find the Difference of Latitude between X and q as follows.

[Rad

[Rad | Coſin. 18.36 || 156 Mi. = Xq | 147′,85 = 2°.27′.51″ Difference of Latitudes.

Coſin. 18.36.	997670	39 .37.	
Log. 156	2 19312	42 . 4.51″	Comes for Pq, inſtead of
	2.16982 Log. 147′,85.	42 . 5.24	Found by the Calculation at large for PQ.
		0 . 0.33″	Error.

And this degree of Exactneſs is undoubtedly ſufficient.]

Or elſe you may ſay

Sin. 77.50 | 156 || Sin. ZXq | Zq = 89,275.
33.55

77. 50.	0.01099		
156.	2.19312	Rad | Coſ. 49.39 || Zq | Diff. of Lat. between Z and q.	57′48″
33. 55.	9.74662		41. 7.
	1.95073	Log. 89,275 = Zq.	Pq = 42. 4.48
Coſ. 49. 39.	9.81121		PQ = 42. 5.24
	1.76194	Log. 57,802	Error c. 0.36

EXAMPLE. III.

Suppoſe now that having made at Sea the very ſame Obſervations of the Sun as in the firſt Example, your Ship, in the mean time between the Obſervations, did ſail by a Loxodromic Courſe, 233,09 Miles in length, making with the Meridian an Angle of 59°; ſteering thus almoſt North Eaſt and by Eaſt, along the Line SQ, Fig. II. It is required to find the Latitudes of the Points S and Q where your Ship was at the Times of the Obſervations; and likewiſe the Difference of Longitude between thoſe two Points; and the true Hours of the Sun, or the Angles APS, BPQ when the Obſervations were made.

To reſolve this uſeful Problem, and the others like it, we may take the following Steps. But here if I give an Example wherein I ſuppoſe ſo long a Run as 233,09 Miles, and ſo long an Interval between the Obſervations as 3¼ Days, I hope it will appear there is no Preſumption in this, ſince I ſhall make a right Uſe of it: And I am ſatisfied that Sea Jewel-Watches or Clocks may be made or rendered ſtill more accurate for Naval Uſes, than they may now ſeem to be; and that there are Methods for making much accurater Obſervations at Sea, and a ſurer Eſtimation of the ſwiftneſs of a Ship, than could be procured by any ordinary Way. Beſides, this Example is taken from Nature. For it will but too often happen that Navigators will have no better Obſervations to work upon, eſpecially in Winter-time; and will think themſelves happy, if they can but have them, and know what to do with them, but chiefly when they can have their ſecond Obſervation not too far from Noon.

I. The firſt Step to reſolve this Problem, is to make the ſame Suppoſitions, and the very ſame ſeven or eight Calculations as in the firſt Example, in order to find not only the dormant or borrowed Zenith Z, which would have reſulted from the Obſervations if the Obſervator had been at reſt; but likewiſe all the Arcs and Angles which are given or found by thoſe very Suppoſitions and Calculations. To which may be added the finding of the Difference of Latitude between the Places of the Ship in the firſt and in the ſecond Obſervation.

II. By this means, ſuppoſing the little Triangle SZQ Fig. II. and V. to be a Rectilinear Triangle, ſince it differs but little from one; and ſuppoſing that the Meridian PZ cuts the Line SQ in N: You will have all the Angles in the Triangles SNZ and ZNQ, and by conſequence in the Triangle SQZ alſo. By which means, having the Side SQ, you will find with no great Error, the Sides SZ, ZQ, and the Diſtances of the Parallels of Latitude paſſing through S and Q, from the Pole P or elſe from the Equator.

III. This however is but a near Approximation, which you may correct either by a mechanical Conſtruction, or rather by a further Calculation as follows, or if you will by both.

As your Approximation may beſt direct you, Take either two different Values of PS, or elſe three different Values of PS in an Arithmetical Progreſſion, the common Difference between the Terms of that Progreſſion being ten Minutes, or, if you will, twenty Minutes.

Then for the leaſt and the biggeſt of thoſe Values of PS calculate at once, by the Rule given in *Phil. Tranſ.* N° 219, or in *Sherwin*'s Mathematical Tables, the reſulting Sides PQ, and likewiſe the reſulting Angles SPQ.

IV. Afterwards for the two or three aſſumed Values of PS calculate at once the reſulting Angles APS. Add them to their proper Angles SPQ. And thus the Angles QPB becoming known, find at once the reſulting two or three Sides BQ, which will not much differ from the given Side BZ. When I ſay that theſe two or three ſets of Calculations muſt ſeverally be made at once, I mean only that you will by theſe Means ſpare a part of the Time and Trouble in calculating. For the Tables will preſent to your view at once two or three of the Elements or Numbers which muſt be employ'd in your Calculations.

V. Theſe Preparations being once made, you will eaſily determine the true Numbers reſulting for the Complements of your Latitudes at the Times of your Obſervations, or for PS and PQ; and likewiſe for the Angles APS, SPQ and QPB, or for the true Hours when you made your Obſervations, and for the Difference of Longitude between the Places you were in at thoſe very Times, as far as you may depend upon the Truth of your Obſervations.

I. Now then to proceed in our Example by the Steps ſet down here; You have already, in your ſeven or eight former Calculations, got over the firſt Step. To which, as an Appendix or a further Preparation, may be added here the finding of the Difference of Latitude between the Points S and Q. For by the Nature of the Loxodromic Line, As the Radius, is to the Coſine of the Angle of your Courſe with the Meridian; ſo is SQ or 233,09 Miles, to the Difference of Latitude between your Ship in S and your Ship in Q: Which Difference is found here to be 2° 5′.

Coſin. of 59° 971184.
Log. 233,09 236752
207936 Log. of 120,05 Miles = 2° 5′.

II. Sin.

II. Sin. SZQ = 102°.10′ | is to SQ = 233,09 || Sin. SQZ = 9° 21′ | SZ = 38′,739
|| Sin. ZSQ = 68.29 | ZQ = 221′,83

	77 .50		
Com. Ar. Sin. 77.50.	0.00987		0.00987
Sin 9.21.	9.21076		2.36752
Log. 233,09	2.36752	Sin. 68.29.	9.96863
	1.58815 Log. SZ = 38′,739		2.34602 Log. ZQ = 221′,83

These Calculations serve to find, by a near Approximation, SZ, ZQ; ZL and Z*L*: And by consequence to find the Latitude of the Points S and Q pretty near the Truth.

R | Cosine 52°.31′ || SZ | ZL.
R | Cosine 49 .39 || ZQ | Z*L*.

Cosine 52.31.	9 78428	Cos. 49.39.	9 81121
Log. SZ	1 58815	L. ZQ	2 34602
	1.37243 L. ZL = 23′,574 = 23′½.		2.15723 L. Z*L* = 143′,62 = 2° 23′½
PZ =	41. 7	PZ =	41. 7
ZL =	23½	Z*L* =	2.23½
PS = PL =	= 40.43½	PQ = P*L* =	38.43½

So then the Difference of Latitudes L*L* which ought to be 2° 3′ comes forth only 2°. Which however intimates a pretty near Approximation.

III. I take then for the three different Values of PS, successively, 40°.30°, and 40°.40′, and 40° 50′. And by the first and last of them I find the Angles SPQ as follows:

PS = 40.30	20.15	Tang.	956693
*L*L = 2. 3			
PQ = 38.27	19.13½		954248
	244,5	Log.	2.38614
	Log. Tang. 59′.		10.22123
			12.60737
	Constant Log.		10.10151
	Log. 320′,52		2.50586

So then SPQ = 5° 20′½.

PS = 40.50	20.25	Tang.	957081
2. 3	½		
PQ = 38.47	19.23½		954653
	242,8	Log.	2.38525
			10.22123
			12.60648
			10.10151
	Log. 319,87		2.50497

So then SPQ = 5°.20′

Then I find the Angles APS and QPB as follows:

Sin. Vers. APS?		930430	APS = 37.1		
Sin. AP,	108.13	997767	SPQ =	5.20½	
Sin. PS,	40.30	981254	QPB =	37.58½	
		909451	12431	80.20	
Sin. Vers. Nat. 67.43			62081		
Sin. Vers. Nat. 75.14			74512		
Sin. Vers. APS?		931211	APS = 37.22		
Sin. AP,	108.13	997767	SPQ =	5.20¼	
Sin. PS	40.40	981402	QPB =	37.37¾	
		910380	12700	80.20	
67.33			61812		
75.14			74512		
Sin. Vers. APS?		931971	APS = 37.42		
Sin. AP,	108.13	997767	SPQ =	5.20	
Sin. PS	40.50	981549	QPB =	37.18	
		911287	12968	80.20	
67.23			61544		
75.14			74512		

And here I find the Sides BQ as follows, in the Triangles QPB.

Sin. Vers. QPB,	37.58½	932576		
Sin. PB,	109.12	997515		
Sin. PQ,	38.27	979367		
		909458	12433	
Sin. Vers. Nat. 70.45			67031	
Sin. Vers. Nat. BQ			79464	78.9 = BQ
Sin. Vers. QPB,	37.37¾	931811		16′ Difference
Sin. PB,	109.12	997515		
Sin. PQ,	38.37	979526		
		908852	12261	
Sin. Vers. Nat. 70.35			66756	
Sin. Vers. Nat. BQ			79017	77.53 = BQ
Sin. Vers. QPB.	37.18	931075		15′ Difference
Sin. PB,	109.12	997515		
Sin. PQ,	38.47	979684		
		908274	12099	
Sin. Vers. Nat. 70.25			66482	
Sin. Vers. Nat. BQ			78581	77.38 = BQ

Now if BQ be made of 77° 41′ as it was observed, you will have proportionably, by the preceeding Calculations;

QPB = 37° 22′; which answers to $2^h\ 29^m\ 12^{sec}$. And this gives the Hour when your last Observation was made, *viz.* 9^h 30′ 48″ in the Morning.

SPQ = 5° 20′; which gives your Departure or Change of Longitude.

APS = 37° 38′; which answers to $2^h\ 30^m\ 32^{sec}$. for the Time of the Day when your first Observation was made in the Afternoon.

PS = 40° 48′. And by consequence 49° 12′ will be your Latitude in the first Observation.

PQ = 38°.45′. And by consequence 51° 15′ comes forth for your Latitude in the second Observation.

Notwithstanding the length of your Way, that is of SQ, and your great Distance from the Equator; you might nevertheless have spared the two middlemost Calculations, which here only justify that the Variations of BQ and of the Angle APS are gradual and very regular.

And

And here, by an Inspection of a Figure regularly made, and fitted to your Example, you are hitherto left to judge what Uncertainties you may be at, whether in reference to your Watches, or to the Length and Position of your Way; and how much those Uncertainties may affect either your Latitudes, or your Difference of Longitude between the Places of your Ship in S and Q. But as all these things have their Bounds, which it has been my Application for these many Years still to reduce within a small Compass, so we can be, especially with those Helps I would make use of, but a little mistaken in reference to the Latitude, though we may be more mistaken in reference to the Longitude.

But the sequel of your Observations will further help to set you right, as to those remaining small Doubts or Errors; whether you can come to a Meridian Observation; or only come to an Observation not very different from Noon; or else procure two proper Observations within a shorter Distance of Time from one another, suppose within less than one Day: Which are the means of avoiding, as to the Latitude, the greater Errors that might arise from the Irregularities of your Watches or Clocks.

EXAMPLE IV.

Suppose, for instance, that on the seventh of *November* you took, among the Clouds, a second Observation of the Sun, when it was come to the Point *a*, Fig. IV. and found its true Distance *az*, from *z* the new resting ~~Zenith~~ 70°. 5′, or else 70°.35′, just $2^h\,25^m\,14^s$, after the other Observation was made in the Morning: And that, having had no Opportunity to follow the Sun any longer, you could pronounce nothing about your nearness to or remoteness from a Meridian Observation. For it is to be noted; that, because of the former *Data* in the preceeding Observations, I may here chuse *az* in Conformity or in Disagreement to the same. The Conformity would be greater if I did chuse *az* = 70°.33′ or there about. But the very Disagreement in the result of the Calculations will be more instructive, and give room to some Reflexions, which it will be proper to lay before you. Fig.

Therefore drawing the new Meridian P*z*, you will have, by reducing the Time, *viz.* $2^h\,25^m\,14^s$, into Degrees, the Angle *a*PB = 36°.18′.30″. PB = 109°.12′, as before. P*a* = 109°.13′.

And you will find B*a* = 34.13½ as follows:

I. Sin. Vers. BP*a* =	36.18.30	928816		
Sin. BP	109.12	997515		
Sin. P*a*	109.13	997510		
		923841	Log.	17314½
Differ.	0. 1	2 Char.		0
Sin. Vers. *a*B = 34.13½				17314½

II. To find the Angle PB*a*, work thus.

Sin. 34.13½ \| Sin. 36.18.30 \|\| Sin. 109.13 \| Sin. PB*a*.

Comp. Ar. Sin.	34.13½	0.24994
Sin.	36.18½	9.77242
Sin.	109.13	9.97510
		9.99746 Sin. 83.49
		96.11 = PB*a*.

III. To find *a*B*z* and PB*z*.

Sin. Vers. *a*B*z*		984572.	72.36	*a*B*z*.
Sin. B*z*	77.41	998989	23.35	PB*z*.
Sin. B*a*	34.13½	975006	96.11	PB*a*.
		958567	38519	
Nat. Vers. Sin. 43.27½			27416	
Nat. Vers. Sin. 70. 5 = *az*			65935	

IV. To find P*z* and consequently the Lat. also of the Point *z*.

Sin. Vers.	23.35	8 92179		
Sin. BP.	109.12	9 97515		
Sin. B*z*.	77.41	9 98989		
		8.88683	7706	
	31.31	9 Ch.	14751	
Nat. Vers. Sin. P*z*			22457	39°. 9′ = P*z*.
				50 .51 Latit. of *z*.

V. To find BP*z* and *a*P*z*.

Sin. Vers. BP*z*?		933142.	38.14	BP*z*.
Sin. PB.	109.12	997515.	36.18½	BP*a*.
Sin. P*z*	39. 9	980027.	1.55½	*a*P*z*.
		910684	12789	
Sin. Vers. Nat. 70. 3			65880	
Sin. Vers. Nat. B*z*. 77.41			78669	

VI. To find P*z*B.

Sin. B*z*.	77.41	gives	0.01011
Sin.	38.14 BP*z*		9.79160
Sin. PB.	109.12		9.97515
			9.77686 Sin. 36.44½ +
			P*z*B = 143.15½ −

VII. To find B*za*.

Sin. Vers. B*za*?		925269	34.48½ = B*za*.
Sin. B*z*	77.41	998989	143.15½ = P*z*B.
Sin. *az*	70. 5	997322	178. 4 = P*za*.
		921580	16436
Sin. Vers. Nat.	7.36.	7 Ch.	878
Sin. Vers. Nat. B*a*, 34.13½			17314

VIII. But to find likewise P*za* directly.

Sin. *az*,	70. 5	0.02678 Comp. ar.
Sin. *a*P*z*,	1.55½	8.52622
Sin. P*a*,	109.13	9.97510
		8.52810 Sin. 1.56

or of 178. 4 = P*za*: Which coming forth as we did find it before, may serve for a Verification.

Thus the Decussations or Angles in *z* of the three great Circles, and, by consequence, of the two small Circles that fall here under consideration, being given, we may make a new Figure, in order to proceed by an Approximation.

Suppose then that your Ship, following still the same Course with an Inclination of 59° to the Meridian, did, between the two last Observations of the Sun in B and *a*, Fig. IV. make a Run of 8,4 Miles or Minutes, represented Fig. VI. by QD: And your Numbers or Elements will result as in the Figure VI, where the Two small Circles aforesaid are Q*z*K and *z*D. Fig.4 Fig.5

For the preceeding Calculations give you the Angles P*z*K or Q*z*L, 53.15½. K*z*D, 34.48½. D*z*L, 91.56. Q*z*D, 145.11½. *z*QD, 5.44½. QD*z*, 29.4.

So

So then refolving the Triangle QDz, you have thefe Analogies, and by them the Lengths of Qz and Dz.

Sin. $145.11\frac{1}{2}$ | 8,4 || Sin. 29.4 | Qz = 7,15.
$34.48\frac{1}{2}$ | 8,4 || Sin. $5.44\frac{1}{2}$ | Dz = 1,47.

	0.2436		0.2436	
Log. 8,4	0.9243		0.9243	
Sin. 29.4	9.6865	Sin. $5.44\frac{1}{2}$	8.9997	
	0.8544	Log. Qz = 7,15	0.1676	Log. Dz = 1,47.

Pz = 39°. 9′
zL = $4\frac{1}{4}$
PL = PQ = 39 . $13\frac{1}{4}$
P*L* = 39 . 8. 57.

Likewife by thefe other Analogies you will find zL and z*L* as follows.

Rad | Cofine QzL = $53.15\frac{1}{2}$ || Qz | zL = 4,277
Rad | Cofine PzD = 88. 4 || Dz | z*L* = 0,0496

Log. Qz	0.8544	Log. Dz	0.1676
Cof. $53.15\frac{1}{2}$	9.7768	Cof. 88.4	8.5281
	0.6312 Log. zL = 4′,277		$\bar{2}$.6957 Log. z*L* = 0.0496 = 0′.3″

Now PL or PQ which is here 39° $13\frac{1}{4}$, was found by the former Calculation 38° 45′: Which Numbers differ by $28'\frac{1}{4}$.

However, as the Latitude of the Point D is determined as exactly as it can be had, becaufe of the Nearnefs of the Obfervation to the Time of Noon, we have little or nothing to change in it.

Neither can we change much the Latitude of the Point Q as determined here. For the Interval of Time being only $2^h\ 25'\frac{1}{4}$, and QD being fo fhort as 8,4 Miles; whether we confider the Uncertainty of its Length, or the Uncertainty of its Inclination to the Meridian, we can hardly fuppofe that Uncertainty to amount to $1\frac{1}{4}$ Minutes in the determination of the Latitude; which might reduce PQ to 39° 12′. The Uncertainty in the Altitudes may be a little greater: But, in the Practice, it will prove ftill more inconfiderable, if the Altitudes be taken by my Method. As to the Currents if there be any, I fuppofe you would do what lies in you to know them, that you might account for them, by increafing or diminifhing proportionably the Length of your Courfe, and its Inclination to the Meridian.

The Uncertainty of the Time fpent in defcribing the Line QD is likewife inconfiderable, both becaufe of its being fo fhort, and becaufe an Error of fome Minutes in that Time, wou'd alter very little the Latitude of the Point D, that Point falling here fo near the Meridian.

We muft then refer the main of thefe Errors to the
Fig. 5. former Vth Figure, where we conftrued the Line SQ. If that Line SQ had been conftrued as the Refult of two Obfervations made in the fame Day, or made, the one in the Afternoon of one Day, and the other in the Morning of the next Day, and not fo late as three or four Days after, as we did heretofore fuppofe: Then the Errors arifing from any of the Uncertainties aforefaid would be fo much the fmaller, and would fhew themfelves fuch in your Calculation.

Fig. 6. But here, as the Latitude of the Points D and Q Fig. VI. was afcertained by the fubfequent Obfervation made in D, and this notwithftanding the great Length of SQ: So, on the other hand, the Latitude of the Point S ought to be afcertained by your Anterior Obfervation made before that other which anfwers to the Point S. For if the fame Day that you made, or one or two Days before you made, the Obfervation S, you departed from a Harbour, or were in a Place, whofe Latitude you know, or if you took then a Meridian Obfervation, or elfe if you took an Obfervation which fell not far from Noon; or laftly, if you took then any Obfervation to which you may compare the Obfervation made in S; Then you may, *mutatis mutandis*, know the Latitude of S with a fufficient Exactnefs, by the fame Method as we have here determined the Latitude of Q. Likewife your Knowledge of the Latitude of the Point D gives you a good Foundation, upon which you may afterwards build your next Obfervations and Calculations.

Having thefe notable Helps, you will be enabled to pronounce more fafely concerning the Longitude of the Points S, Q, D; efpecially comparing alfo the Hours of your Obfervations as found by your Watches or Clocks, with the Hours found by your Calculations. And in this the beft Watches will have the Advantage of thofe that are not fo good: And my Sea Jewel-Watches will fhew what may be expected from them; which I muft leave to Experience. But this by the by: For at prefent what I aim at in this Difcourfe is chiefly the Latitude; on whofe Knowledge our very Sailors make, in a great meafure, to depend their imperfect Guefs at the Longitude.

As to the Angle QPD, which gives the Difference of Longitude between the Points Q and D, as refulting from the given Length of QD = 8,4: You need here take no further Trouble about it, that Line being fo fhort, but only to make ufe of this Proportion fufficiently accurate for the Purpofe, *viz.* SQ, is to the Angle SPQ; or 233,09 Miles, to 5° $20'\frac{1}{2}$: as QD or 8,4 Miles, to the Angle QPD = 11′34″.

In the VIII Figure, fuppofe the Meridian Altitude was taken in O in the Parallel OE, three Days before the third of *November*; And that the fame Day, *viz.* the 31ft of *October*, in the Morning, the Sun's Altitude was likewife taken when the Ship was in H, and, if you will, in the Afternoon alfo, when the Ship was in K. Fig. 8

If your Ship did at firft fet out from C, defcribing in given Times the Lines CH, HK, KS, SQ, QD, whofe Proportion is given, as far as your Eftimation can approach the Truth; The very Proportions between thofe Lines, efpecially between OS and SQ, will give a great light for determining better the Pofition or Obliquity and Length of the Ship's Way; and the Difference of Longitude between O and D, or between C and D.

If, inftead of Two Altitudes of the Sun, we have Two Altitudes of a Star, or of two Stars, or of the Sun and a Star, or even if we have One or Two Altitudes of fome other Planets inftead of the Sun, The fame Method will ferve to find the Latitude at Sea. Only it muft be remembred that the diurnal Revolution of fixed Stars thro' 360 Degrees does amount, not to 24 Hours, but to $23^h\ 56^m\ 4^{sec}$. The diurnal Revolution of other Planets may be derived from printed Ephemerides.

This however I have no defign to treat of at prefent. Many others can do it at leaft as well as myfelf. And, before I publifh what I have further in ftore for the Improvement of Navigation, I muft fee how that will be received which I now publifh.

Thus you have here the Refolution of the Problem which we did propofe. You have it, on one Side, as accurate as the niceft Calculators can defire. And as for them that chufe to work, as much as may be, by Figures and Inftruments, you have here a Profpect of what may be done in that kind, whether they will work altogether by Figures, or rather work partly by Calculations alfo.

A fuccefsful Experiment of this Method was tried by me, in *January* $171\frac{4}{5}$. We were coming from the Straights, aboard the Duke, Captain *Nathanael Clerk* Commander; and for feveral Days could take no Meridian Altitude. But one Day I promifed him, from any Two convenient Altitudes of the Sun, taken among the Clouds, to find the Latitude: And accordingly, by Two Obfervations taken in the Morning, I did it, in perfect Agreement with what they did obferve at Noon. He took a particular care that I fhould know nothing of the Meridian Altitude which they did find. And, in order to try my

my Method the better, he did even tell me, before I had gone through my Calculations, that we were in a Latitude different from ours by one whole Degree. But when I had done, I told him confidently, to his great Surprize, Captain, You ſaid we were in ſuch a Latitude: But I tell you we are in ſuch another Latitude; naming thoſe very Degrees and Minutes which himſelf and others had obſerved.

WAVES, in the open Seas, are exceeding regular, and may ſerve, as I have ſaid, for a ſort of Time-keeper. Their Number, as they paſs under a Ship, may be continually mark'd of itſelf, by an Inſtrument which I call a *Kymometer*, and which will hereafter be of excellent Uſe for Navigation.

By this means, Two Ships being at firſt on the ſame Wave, and obſerving then the Numbers mark'd by their Kymometers, or cauſing them to agree, may know afterwards (from as far as they can hear and ſee any Signals given by One Ship to the Other, or any Signals given by a Third Ship placed in or near the Line that joins them both) How many Waves there is, at any time, between the Ships,

And when 1000 or 2000 or 3000 Waves, *&c.* have paſſed under the Ship, if a great Gun be fired once or ſeveral times, by one of the two Ships, or by the middlemoſt of the three Ships, at the Inſtant of their giving a Signal to the other or to both the other Ships; The Obliquity of the Waves to the Lines that join the Ships being obſerved alſo: Then the Length of the Time obſerved, between the Moments of firing the Guns and of hearing their Report, will be ſufficient to calculate thereby the Breadth of each Wave.

And if one of the Ships remains in the ſame Place, or i the ſame unmoveable Line parallel to the Waves themſelves, The Swiftneſs of the Waves will be known alſo; and that ſo much the better, as the Line joining the Ships is leſs oblique to the Waves.

The Swiftneſs of Sounds has been obſerved nicely by the ~~late~~ Mr. *Derham* R.S.S.

The Figure of Waves is very regular alſo. It ſeemed to me in the Ocean to repreſent the Line of Sines; where the Degrees of a Circle being reckoned on the Horizontal Abſciſſa perpendicular to the Waves, the Ordinates perpendicular to the Horizon repreſent for each Degree their correſpondent Sines.

And ſo, The very ſame Figure which the upper Part of the Wave has above the Natural Level of the Sea; the very ſame Figure, I ſay, has alſo ſenſibly, in an inverted Situation, the concave Part of the Wave, under the natural Level of the Sea.

In coming from the Straights in the Ship the Duke, when the Wind was ſaid to be a brave Gale, becauſe it was favourable to us, but would have been called a Storm by Ships to whom that Wind was contrary; Each Wave, as we ſailed along, did paſs under our Ship, exactly in ſix Seconds of Time. I did not meaſure their Bigneſs; but I am ſatisfied they look'd and were much bigger, than according to Sir *Iſaac Newton*'s Theory.

The Swiftneſs of the Waves was then ſuch, that the Top of moſt of them would juſt begin to offer to break, but could only ſhew a ſmall Whiteneſs rowling on the Top of thoſe Waves, keeping an equal pace with the Waves themſelves.

In the Theory of Waves, the *Vis Centrifuga* of the Parts of Water muſt be conſidered, both when the Curves which they deſcribe turn their Convexity towards Heaven, and alſo when they turn it towards the Earth. And the various Depth of the Sea muſt be conſidered alſo. It is that *Vis Centrifuga* that does juſt ſupport thoſe ſo prodigious Maſſes of Water above the natural Level of the Sea. And it is the like Force, in a contrary Poſition and Degree, that does dig continually, between the Waves, thoſe prodigious Furrows, under the ſame natural Level of the Waters: In which Level the Waves have their *Punctum Flexûs contrarii.*

Theſe few Remarks will be ſufficient for Others to eſtabliſh, as I have done elſewhere, the Theory of Waves upon a Mathematical Foundation: To which it will remain to give the laſt Hand, by a long and accurate Experience.

The Librations of Water in a Tube, as propoſed by Sir *Iſaac Newton*, have ſcarce any Reference to the real Theory of Waves: Both becauſe they have no Relation with any *Vis Centrifuga*; And becauſe the Waves working on every Part of the Sea which is in reach of their Agitation, the infinite Perplexity of thoſe imagined Tubes, and their Penetration with one another, would confound all their Operations, and all manner of Mathematical Reaſonings about them.

F I N I S.

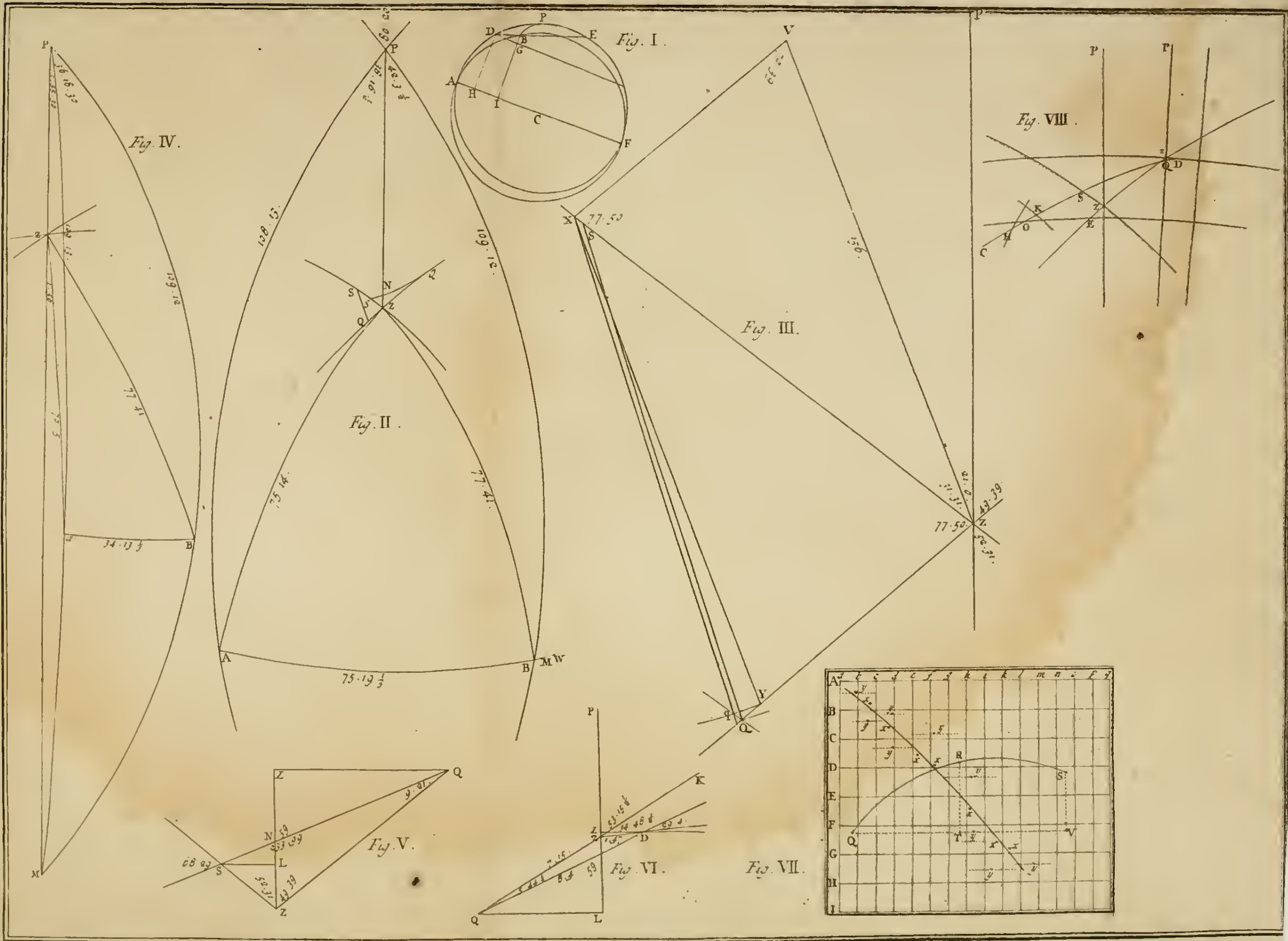

Fig. I.
Fig. II.
Fig. III.
Fig. IV.
Fig. V.
Fig. VI.
Fig. VII.
Fig. VIII.